Exercícios Aula 61 - Apostila 3

① O valor de $\cos 72° - \cos^2 36$ é idêntico ao de:

a) $\cos 36°$ b) $-\cos^2 36$ c) $\cos^2 36°$ d) $-\text{sen}^2 36°$ e) $\text{sen}^2 36°$

Como todas as alternativas estão utilizando o ângulo de 36°, devemos reescrever o cos 72°.

Assim, utilizando a fórmula da multiplicação de arcos, teremos:

$$\cos 2x = \cos^2 x - \text{sen}^2 x$$

$$\cos 72° = \cos(2.36°) = \cos^2 36° - \text{sen}^2 36°$$

Voltando à expressão dada, teremos:

$$\cos 72° - \cos^2 36° = \cancel{\cos^2 36°} - \text{sen}^2 36° - \cancel{\cos^2 36°} =$$

$$= -\text{sen}^2 36°$$

ALTERNATIVA D

② O valor de $tg\,10^\circ(sec\,5^\circ + cosec\,5^\circ)(cos\,5^\circ - sen\,5^\circ)$ é igual a:

a) 2  b) $\frac{1}{2}$  c) 1  d) $\sqrt{2}$

A maioria dos ângulos da expressão é 5°, escreveremos a tangente de 10° de forma a utilizar 5°, e sec e cosec escrevemos utilizando cos e sen.
Então:

$$tg\,10^\circ(sec\,5^\circ + cosec\,5^\circ)(cos\,5^\circ - sen\,5^\circ) =$$

$$\frac{sen\,10^\circ}{cos\,10^\circ}\left(\frac{1}{cos\,5^\circ} + \frac{1}{sen\,5^\circ}\right)\left(\frac{cos\,5^\circ - sen\,5^\circ}{1}\right) =$$

$$= \frac{sen(2.5^\circ)}{cos(2.5^\circ)}\left(\frac{sen\,5^\circ + cos\,5^\circ}{cos\,5^\circ . sen\,5^\circ}\right)\left(\frac{cos\,5^\circ - sen\,5^\circ}{1}\right) =$$

$$= \frac{2\cancel{sen\,5^\circ cos\,5^\circ}}{cos^2 5^\circ - sen^2 5^\circ}\left[\frac{(sen\,5^\circ + cos\,5^\circ)(cos\,5^\circ - sen\,5^\circ)}{\cancel{cos\,5^\circ . sen\,5^\circ}}\right] =$$

$$= \frac{2}{\cancel{(cos\,5^\circ - sen\,5^\circ)(cos\,5^\circ + sen\,5^\circ)}} \cdot \cancel{(cos\,5^\circ + sen\,5^\circ)(cos\,5^\circ - sen\,5^\circ)} =$$

$$= 2$$

ALTERNATIVA A

③ Dado $tg \frac{x}{2} = \frac{1}{2}$, então $sen\, x - cos\, x$ é igual a: obs: considere $0 < x < \pi/2$

a) $\frac{7}{5}$ b) $\frac{4}{5}$ c) $\frac{3}{5}$ d) $\frac{1}{5}$ e) $\frac{2}{5}$

Sendo $tg \frac{x}{2} = \frac{1}{2}$ então $\frac{sen \frac{x}{2}}{cos \frac{x}{2}} = \frac{1}{2}$

$$sen \frac{x}{2} = \frac{1}{2} \cdot cos \frac{x}{2} \Rightarrow \boxed{2\, sen \frac{x}{2} = cos \frac{x}{2}}$$

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Pela relação fundamental, temos:

$$sen^2 \frac{x}{2} + cos^2 \frac{x}{2} = 1$$

$$sen^2 \frac{x}{2} + \left(2\, sen \frac{x}{2}\right)^2 = 1$$

$$sen^2 \frac{x}{2} + 4\, sen^2 \frac{x}{2} = 1$$

$$5\, sen^2 \frac{x}{2} = 1$$

$$sen^2 \frac{x}{2} = \frac{1}{5}$$

$$sen \frac{x}{2} = \sqrt{\frac{1}{5}}$$

$$\boxed{sen \frac{x}{2} = \frac{\sqrt{5}}{5}}$$

Substituindo:

$$2 \operatorname{sen} \frac{x}{2} = \cos \frac{x}{2}$$

$$2 \cdot \frac{\sqrt{5}}{5} = \cos \frac{x}{2}$$

Assim:

$$\operatorname{sen} x - \cos x =$$

$$= \operatorname{sen}\left(\frac{x}{2} + \frac{x}{2}\right) - \cos\left(\frac{x}{2} + \frac{x}{2}\right) =$$

$$= 2 \operatorname{sen} \frac{x}{2} \cdot \cos \frac{x}{2} - \left(\cos^2 \frac{x}{2} - \operatorname{sen}^2 \frac{x}{2}\right) =$$

$$= 2 \cdot \frac{\sqrt{5}}{5} \cdot \frac{2\sqrt{5}}{5} - \left[\left(\frac{2\sqrt{5}}{5}\right)^2 - \left(\frac{\sqrt{5}}{5}\right)^2\right] =$$

$$= \frac{4 \cdot 5}{25} - \left[\frac{4 \cdot 5}{25} - \frac{5}{25}\right] =$$

$$= \cancel{\frac{20}{25}} - \cancel{\frac{20}{25}} + \frac{5}{25} =$$

$$= \frac{5}{25} = \frac{1}{5}$$

ALTERNATIVA D

(4) O valor do produto $\text{sen}\,\frac{\pi}{8} \cdot \cos\frac{\pi}{8}$ é igual a

a) $\frac{\sqrt{2}}{8}$ b) $\frac{\sqrt{3}}{4}$ c) $\frac{\sqrt{2}}{2}$ d) $\frac{\sqrt{3}}{2}$ e) $\frac{\sqrt{2}}{4}$

$$\left(\text{sen}\,\frac{\pi}{8} \cdot \cos\frac{\pi}{8}\right) =$$

$$= 1 \cdot \left(\text{sen}\,\frac{\pi}{8} \cdot \cos\frac{\pi}{8}\right) = \frac{1}{2} \cdot \underbrace{2\left(\text{sen}\,\frac{\pi}{8} \cdot \cos\frac{\pi}{8}\right)}_{\text{sen}\left(\frac{\pi}{8}+\frac{\pi}{8}\right) \text{ ou } \text{sen}\left(2 \cdot \frac{\pi}{8}\right)} =$$

Adição de arcos ou multiplicação de arcos

$$= \frac{1}{2} \cdot \left(\text{sen}\, 2 \cdot \frac{\pi}{8}\right) = \frac{1}{2} \cdot \text{sen}\,\frac{\pi}{4} =$$

$$= \frac{1}{2} \cdot \text{sen}\,45^\circ = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$$

ALTERNATIVA E

(5) Se $\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)=m$ e

$\cos(\alpha+\beta)-\cos(\alpha-\beta)=n$, em que

$0<\alpha, \beta<\frac{\pi}{2}$, então pode-se afirmar que $\frac{m}{n}$

equivale a:

a) $-tg\,\beta$ b) $\frac{\cos\beta}{\operatorname{sen}\alpha}$ c) $-\cot g\,\beta$ d) $\frac{\operatorname{sen}\alpha}{\cos\beta}$

$$\frac{m}{n}=\frac{\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}=$$

$$=\frac{\operatorname{sen}\alpha\cos\beta+\operatorname{sen}\beta\cos\alpha+\operatorname{sen}\alpha\cos\beta-\operatorname{sen}\beta\cos\alpha}{\cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta-(\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta)}=$$

$$=\frac{\operatorname{sen}\alpha\cos\beta+\operatorname{sen}\alpha\cos\beta}{-\operatorname{sen}\alpha\operatorname{sen}\beta-\operatorname{sen}\alpha\operatorname{sen}\beta}=$$

$$=\frac{2\operatorname{sen}\alpha\cos\beta}{-2\operatorname{sen}\alpha\operatorname{sen}\beta}=-\frac{\cos\beta}{\operatorname{sen}\beta}=\boxed{-\cot g\,\beta}$$

ALTERNATIVA C